

Can Dark Energy emerge from quantum effects in compact extra dimension ?

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ABSTRACT

The origin of the observed acceleration of the expansion of the universe is a major problem of modern cosmology and theoretical physics. Simple estimations of the contribution of vacuum to the density energy of the universe in quantum field theory are known to lead to catastrophic large values compared to observations (Weinberg 1989). Such a contribution is therefore generally not regarded as a viable source for the acceleration of the expansion. In this letter we propose that the vacuum contribution actually provides a small positive value to the density energy of the universe. The underlying mechanism is a manifestation of the quantum nature of the gravitational field, through a Casimir-like effect from an additional compact dimension of space. A key ingredient is to assume that only modes with wavelength shorter than the Hubble length contribute to the vacuum. Such a contribution gives a positive energy density, has a Lorentz invariant equation of state in the usual 4D spacetime and hence can be interpreted as a cosmological constant. Its value agrees with observations for a radius of a 5th extra dimension given by $35\mu\text{m}$. This implies a modification of the gravitational inverse square law around this scale, close but below existing limits from experiments testing gravity at short range (Adelberger et al. 2009).

Key words. Casimir effect - dark energy - cosmological constant - extra dimension

1. Introduction

The evidence for the acceleration of the expansion of the universe has gained in strength since the first result from the Hubble diagram of distant type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1999). The angular power spectrum of the fluctuations in the cosmic microwave background and the large scale properties of the galaxy distribution are all consistent with the accelerated expansion of an homogenous universe, while no alternative Friedmann-Lemaître model seems to be able to reproduce these three data sets (Frieman et al. 2008; Blanchard 2010). Dark energy, the origin of the cosmic acceleration, is often qualified as one of the deepest mysteries of modern physics whose origin is hard to explain within the standard framework of high energy physics (Weinberg 1989). This issue is a tremendous stimulation for the community, producing a rich ensemble of theoretical approaches, while being the target of unprecedented efforts in astrophysical observational strategy,

either in the form of ground projects (LSST Science Collaborations et al. 2009) or ambitious space projects like EUCLID (Laureijs et al. 2011).

A genuine Cosmological Constant Λ , as introduced by Einstein in 1917 (Einstein 1917), accounts for the observed cosmic acceleration. However, most scientists agree on the lack of theoretical motivation for the introduction of such a term into the Einstein equations (Amendola et al. 2012). One reason that is often invoked is that Λ introduces a new fundamental energy scale if one introduces the Planck constant \hbar :

$$E_\Lambda = \left((\hbar c)^3 \rho_\Lambda \right)^{1/4} \simeq 2 \text{ meV} \quad (1)$$

with $\rho_\Lambda = \Lambda c^4 / (8\pi G)$ representing 73 % of the energy content of the Universe. At this energy scale, no exotic physics is a priori expected. Equivalently, a dimensional length scale can be associated:

$$\ell_\Lambda = \left(\frac{\hbar c}{\rho_\Lambda} \right)^{1/4} \simeq 83 \text{ } \mu\text{m} \quad (2)$$

An experimental effort is devoted to observe any deviation from the gravitational laws (Adelberger et al. 2009) at this scale and below. No anomaly has been observed just below this length scale (Kapner et al. 2007). Of course, as we are going to see, this does not exclude any deviation in the gravitational laws controlled by ℓ_Λ , because numerical factors could lower the true length scale to a value smaller than ℓ_Λ . Finally, let us mention that if Λ is a true fundamental constant, then from \hbar , c , G and Λ it is not possible to define a single natural length scale, but instead one can have

$$\ell = \Lambda^{-1/2} f \left(\frac{\hbar G \Lambda}{c^3} \right) \quad (3)$$

with f an arbitrary function of the dimensionless constant $\frac{\hbar G \Lambda}{c^3} \sim 2 \times 10^{-122}$. This small number is nothing else than a reformulation of the cosmological constant problem. Taking $f(x) = 1$ leads to a cosmological scale for ℓ (the size of a static Einstein Universe), $f(x) = \sqrt{x}$ leads to the Planck length while $f(x) = x^{1/4}$ gives the previously introduced scale ℓ_Λ , qualified in the literature as the natural Dark energy length scale (note that this scale is the geometric mean of the two former scales).

Historically, a physical explanation for the Cosmological constant came from the identification of this term with a Lorentz invariant vacuum (Lemaître 1934), which leads to the possibility of a gravitationally active vacuum due to the contribution of zero-point energy. This attractive idea has been discussed as early as in the 1920s by Nernst and Pauli (see Straumann (2002); Peebles & Ratra (2003); Kragh (2011) for a historical presentation) but it was immediately realized that this possibility is plagued by a large discrepancy in estimate order of magnitude. In order to avoid dramatic consequences for cosmology, it is usually assumed that those vacuum energies do not gravitate or give a renormalized value which is exactly zero (this is the first cosmological constant problem). We are therefore left with the second cosmological problem, that is to say how to explain the small “incremental” positive value observed today. A first original idea has been historically proposed in Zel’dovich (1967); Zel’dovich (1968), considering ρ_Λ as being gravitational interaction energy between virtual pairs of the QED vacuum. Unfortunately, this elegant proposition could still not explain the low value of the possible cosmological constant. Nowadays, the actual contribution of vacuum to the present day density of the universe is still the subject of debate in the scientific community.

In this letter, we focuss on the possibility to identify the cosmological constant with effects from the quantum vacuum by considering spatial compact extra-dimensions. Indeed, pioneer papers in the 80s (Appelquist & Chodos 1983; Appelquist & Chodos 1983; Rohrlach 1984) have computed the quantum corrections in the energy density of vacuum stemming from the presence

of such extra dimensions. It has been shown that those quantum corrections correspond in fact to a Casimir effect of the gravitational field induced by the periodic conditions along the extra compact dimensions. Later identification of this quantum correction with the Cosmological constant (Milton 2001; Elizalde 2006b) unfortunately failed because of the wrong sign of the corresponding cosmological constant. In the following, it is shown that the inclusion of the Hubble horizon scale as a maximal wavelength allowed for quantum vacuum modes provides a mechanism to generate a positive cosmological constant.

2. The zero-point energy contribution to the vacuum

Considering the example of a massive scalar field, the contribution of zero-point energy to the density can be obtained as the vacuum expectation value of the 00 component of the energy momentum tensor $T^{\mu\nu}$ ($\hbar = c = 1$)

$$\rho_v = \langle 0|T^{00}|0\rangle = \int \frac{d^d \mathbf{k}}{(2\pi)^d} \frac{1}{2} \sqrt{k^2 + m^2} \quad (4)$$

with d the number of spatial dimension and k the wave vector. The vacuum pressure can be computed in a similar way to the density

$$p_v = (1/3) \sum_i \langle 0|T^{ii}|0\rangle = \frac{1}{d} \int \frac{d^d \mathbf{k}}{(2\pi)^d} \frac{1}{2} \frac{k^2}{\sqrt{k^2 + m^2}} \quad (5)$$

These contributions are highly divergent and therefore need some regularization treatment. The most trivial regularization procedure would be to introduce an ultraviolet cutoff k_c in momentum above which the theory breaks down. Nevertheless, this procedure introduces two flaws : i) the energy density scales as k_c^{d+1} , which leads to a catastrophic value compared to the observed energy density in our universe for any scale k_c related to high energy physics scales, ii) this cutoff in momentum explicitly violates Lorentz-invariance and leads to a vacuum expectation value of the energy-momentum tensor which is not proportional to $g_{\mu\nu}$ and therefore cannot be accepted as such for a description of vacuum. The inclusion of non-Lorentz invariant counter terms can restore the symmetry and lead to the correct equation of state (Hollenstein et al. 2011). Another convenient approach is to use a covariant regularization, such as the dimensional regularization in which the number of dimensions d is written as $d = D + \epsilon$, with D an integer and $\epsilon \rightarrow 0$. Introducing a constant μ (the dimension of which being a mass, or the inverse of a length) so that the energy density and pressure keep the correct dimension, one obtains (see for example Martin (2012))

$$p = -\rho = \frac{m^{d+1} \Gamma\left(-\frac{d+1}{2}\right)}{\mu^\epsilon 2^{d+2} \pi^{\frac{d+1}{2}}} \quad (6)$$

For instance, for $D = 3$, discarding the diverging $1/\epsilon$ term and using the modified minimal subtracting scheme, one finally obtains

$$p = -\rho = -\frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right) \quad (7)$$

It is now explicit that the Lorentz-invariance is preserved (since $p = -\rho$). Moreover, the scaling of the energy density is now like m^{d+1} , which is better than k_c^{d+1} in the hard cutoff regularization. Nevertheless, the presence of the regulator μ does not allow a prediction for ρ while natural values for μ leads to catastrophic large value compared to the observed value of ρ_Λ . In any event, the important point to be stressed at this point is that for a massless field ($m = 0$) the contribution to the vacuum energy density is exactly zero so that this regularization procedure accounts for a degravitation of massless fields, even if it does not give any physical mechanism that would be at its origin (see Ellis et al. (2011); Smolin (2009) for one example of such theories). This result

corroborates the simple remark that if we were to build a traceless energy momentum tensor from the metric $g^{\mu\nu}$, the only solution is to have $\langle T^{\mu\nu} \rangle = 0$. Say differently, in order to accomodate the equation of radiation (ie massless fields), $p = \rho/d$ with the one of vacuum, $p = -\rho$, one needs $p = \rho = 0$.

Though the specific consideration for a massless field does not stand for a general demonstration, the previous consideration corroborates the standard conclusion that some mechanism sets the contribution of vacuum energy to exactly zero in an isotropic spacetime of arbitrary dimensions. The origin of the acceleration of the expansion of the universe is then logically expected to happen from a distinct physical mechanism. The late domination of a scalar field or modifications to the Einstein-Hilbert action are the two options most investigated by now, subject of intensive research activities since the evidence for an accelerated expansion (Clifton et al. 2012; Tsujikawa 2010).

In the next section we present a physical mechanism to generate a non-zero positive density energy and pressure from zero-point energies of a massless field (the gravitational field itself). This is achieved by assuming the existence of an additional compact spatial dimension, which will therefore modify equation (6). It is well known that a modification of the boundary conditions of a quantum field lead to non trivial physical properties of the vacuum. The Casimir force between two infinite conducting plates is a famous example of a physical non-zero but finite contribution from the QED vacuum even if the electromagnetic field is massless. In the latter configuration, isotropy of space has been obviously broken by the presence of boundary conditions. The pressure in the direction normal to the plates satisfies $p_{\perp} = 3\rho$ (with $\rho < 0$) while the pressure parallel to the plates satisfies $p_{\parallel} = -\rho$ (Brown & Maclay 1969), in accordance with the traceless nature of the electromagnetic field. Remarkably enough, the Lorentz invariance in the 2 dimensions parallel to the plates ensures the equation of state $p_{\parallel} = -\rho$ with a non-zero value of ρ . As we will see, in the presence of additional compact dimensions of space, a gravitational Casimir effect allows for a non-zero density energy which is Lorentz invariant in the usual 4D spacetime ($p = -\rho$), even for a massless (traceless) field.

3. Casimir effect from higher compact dimension

The presence of additional space dimensions has been proposed with various motivations in modern physics (for a review, see for example Rubakov (2001)), from the Kaluza-Klein scenario (Kaluza 1983) aiming at unification of interactions to the more recent braneworld paradigm dealing with the hierarchy issue (Arkani-Hamed et al. 1998; Randall & Sundrum 1999). In this picture, matter is localized in a 4D spacetime (the brane) while gravity can propagate in all the dimensions (the bulk). The gravitational field being massless, dimensional regularization (equation (6)) ensures that the energy density vanishes in arbitrary N dimensional infinite isotropic spacetime. However, in the case of compact additional dimensions, the situation is different since the structure of the quantum vacuum is modified by the quantification of the gravitational field along the additional dimensions. This quantification of the gravitational field modes in the bulk leads to a Casimir energy that has been computed many years ago for one extra dimension in pioneer works from the 80' in Appelquist & Chodos (1983); Appelquist & Chodos (1983) for a Minkowski background metric and later in Rohrllich (1984) using a zero-point energy calculation and an exponential cutoff regularization. Generalization to a spacetime structure $M^4 \times S^N$ has been done in Candelas & Weinberg (1984); Chodos & Myers (1985) for N odd and later in Myers (1986); Kantowski & Milton (1987) for N even.

In Table 1, we summarize expectations and constraints on the size of such extra dimension for different values of the number of extra dimension N . The third column provides the radius that would lead to a vacuum contribution equal to the present dark energy density (the sign being those of the normalization constant in column 2). The fourth column gives the size of the additional dimension that would solve the hierarchy problem (i.e. in order to have a Planck scale equal to 1 TeV). The last column summarizes the present observational constraints on the size of such extra

N	$\kappa_N = \frac{R^4 \rho}{\hbar c}$	R_N^Λ (μm)	$R_N^{\text{hierar.}}$ (μm)	Constraints (μm)
1	-2.5×10^{-4}	(10.5)	2.6×10^{19}	< 44 (ISL) < 44 (NS)
2	—	—	2.2×10^3	< 30 (ISL) < 0.00016 (NS)
3	1.1×10^{-3}	15.0	9.7×10^{-3}	$< 2.6 \times 10^{-6}$ (NS) $< 10^{-3}$ (LHC)
4	—	—	2.0×10^{-5}	$< 3.4 \times 10^{-7}$ (NS)
5	1.2×10^{-2}	27.2	5.0×10^{-7}	$< 1.0 \times 10^{-7}$ (NS)
6	—	—	4.3×10^{-8}	$< 4.4 \times 10^{-8}$ (NS)
7	3.6×10^{-2}	36.1	7.3×10^{-9}	$< 2.4 \times 10^{-8}$ (NS)

Table 1. The first column gives the relation between the gravitational Casimir energy ρ and the radius of the extra dimension in $R^4 \times S^N$ compactification, for different values of N . The second column gives the radius of the extradimension such that this gravitational Casimir energy is equal to the observed Dark Energy energy density. The third column gives the radius that is necessary to solve the hierarchy problem. Finally, the last column give the present-day constraints (Beringer et al. 2012) on the size of such extra dimensions. ISL is for Inverse Square Law test (Adelberger et al. 2009), NS for Neutron Star constraints (Hannestad & Raffelt 2003), and LHC from the CMS experiment at CERN.

dimension, from $N = 1$ to $N = 7$ extra dimensions. For instance one can see from this table that the hierarchy problem can be solved only with $N \geq 6$. One extra dimension ($N = 1$) is excluded, but also up to $N = 5$ extra dimensions. This leads to the conclusion that extra dimensions cannot solve the hierarchy problem and explain the origin of dark energy at the same time. Moreover we see that odd value of N strictly greater than 1 cannot explain the value of dark energy density. Even values of N are more problematic, as the evaluation of their contribution contains a logarithmic term of some unknown scale μ . This makes the normalization constant (column 2) not well determined. Nevertheless, any plausible value of μ (let say below the Planck mass) will not make a large numerical difference and will therefore lead to a radius not very different from those obtained in the odd case. Therefore we are left with the only possibility of having only one extra dimension. However, as can be seen from table 1, a negative sign seems to be then obtained for ρ , while observations request a positive sign. More sophisticated scenarios have been proposed to overcome this dead-end (Cognola et al. 2005; Elizalde 2006a), although no convincing solution has emerged. In conclusion, previous attempts to directly identify this Casimir energy with the cosmological constant unfortunately failed (Milton 2001; Elizalde 2006b).

In what follows, we first show how to reproduce the calculation for $N = 1$ using dimensional regularization. Let us hence assume the existence of one spatial additional dimension compactified around a circle of radii R . The periodic condition $f(x^i, x^4 + 2\pi R) = f(x^i, x^4)$ allows the metric tensor to be expanded in Fourier serie

$$g_{\mu\nu}(x^i, x^4) = \sum_{n=-\infty}^{\infty} g_{\mu\nu}^{(n)}(x^i) \exp(inx^4/R) \quad (8)$$

where x^4 is the coordinate along the extra dimension. In a specific gauge, the metric satisfies the propagation equation $\nabla^2 g_{\mu\nu} = 0$ so that the gravitational modes $g_{\mu\nu}^{(n)}$ satisfy the dispersion relation :

$$\omega_n(\mathbf{k}) = \sqrt{\mathbf{k}^2 + n^2/R^2} \quad (9)$$

The mode $n = 0$ is the usual massless graviton, while the excited modes $n \neq 0$ correspond to effective massive gravitational fields of masses n/R (Kaluza-Klein tower). In order to simplify, we shall modelize the gravitational field by a scalar field, and multiply the final result by the number of polarization states $p_m = m(m-3)/2$ in m -dimensional spacetime ($p_5 = 5$). The previous assumption is justified since we consider a flat extra dimension. For situations with curvature, a conformally coupled scalar field would have been a better description of the true gravitational field.

With a vacuum energy per mode given by $\omega_n/2$, the total vacuum energy density is obtained as

$$\rho = \frac{p_{d+2}}{2\pi R} \sum_{n=-\infty}^{\infty} \int \frac{d^d \mathbf{k}}{(2\pi)^3} \frac{1}{2} \sqrt{\mathbf{k}^2 + n^2/R^2} \quad (10)$$

Each term in the previous sum can be dimensionally regularized using eq. (6) :

$$\rho = -\frac{2p_{d+2}\Gamma\left(-\frac{d+1}{2}\right)}{(4\pi)^{\frac{d+3}{2}}R^{d+2}} \zeta_R(-d-1) \quad (11)$$

where ζ_R is the Zeta Riemann function. Using the reflection formula

$$\Gamma\left(\frac{z}{2}\right)\zeta_R(z)\pi^{-z/2} = \Gamma\left(\frac{1-z}{2}\right)\zeta_R(1-z)\pi^{(z-1)/2} \quad (12)$$

one finally obtains a finite (regularized) contribution for $d = 3$

$$\rho_{\text{App}} = -\frac{15\zeta_R(5)}{128\pi^7 R^5} \simeq -5.1 \times 10^{-5} \frac{1}{R^5} \quad (13)$$

This expression agrees with previous studies based on different regularization schemes (hard cutoff in Appelquist & Chodos (1983) and exponential cutoff in Rohrlich (1984)). Generalizations to N extra dimension with N odd is summarized in table 1 (the differences with Candelas & Weinberg (1984); Chodos & Myers (1985) is the polarization factors p_{d+2}).

In the following we re-examine this question in the cosmological context. We show that this provides a mechanism leading to a positive Casimir energy density ρ at late time. The key ingredient is to take into account the finite age of the Universe. This finite age implies the existence of a length scale, the Hubble horizon $H(t)$. This clearly adds a boundary condition that has to be taken into account. We make two assumptions to account for this effect. First, only modes corresponding to wavelengths shorter than the Hubble length $H(t)^{-1} \sim ct$ contributes to the density of the vacuum energy (see also Cahill (2011) for a similar proposition in the cosmological context). The second assumption is that as long as the Hubble horizon is smaller than the radius R of the extra dimension, the energy density is equal to zero. The reason is that when the horizon is smaller than the radius of the extra dimension, the structure of the quantum vacuum cannot depend on the compact nature of the extra dimension because gravitons have not yet explore the “compactness” of space. The situation should therefore be equivalent to the one previously discussed of a massless scalar field in an isotropic spacetime, leading to $\langle T_{\mu\nu} \rangle = 0$ (see Eq. (6)). It is easy to see why those assumptions can lead to a net positive contribution of zero-point energy. Indeed, when the horizon crosses the radius of the extra dimension, the change in the vacuum is only due to new modes which appear with wavelength larger than $2\pi R$. Those modes contribute with $\hbar\omega/2$ of vacuum energy and an UV cutoff of order $1/R$, leading to a finite positive contribution. In this picture, the cosmological constant can be seen as a “temporal” Casimir effect, as if the boundary conditions were switched on at a given moment of time. The observable quantity being therefore the change of vacuum energy when the horizon crosses the extra dimension.

The previous discussion implies that (13) has to be changed in order to fix the subtraction point in the energy density at $t = R/c$. In order to perform this task, we add to (10) a low energy cutoff $\omega_n(\mathbf{k}) > 2\pi/t$ and a counterterm $CT(t)$ which restores Lorentz invariance and insures that ρ is zero as long as $t \leq R/c$,

$$\rho(t) = \frac{5}{R} \int_{\omega_n(\mathbf{k}) > 2\pi/t} \frac{d^3 \mathbf{k}}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \frac{1}{2} \sqrt{\mathbf{k}^2 + n^2/R^2} + CT(t) \quad (14)$$

with $CT(t)$ such that $\rho(t \leq 2\pi R/c) = 0$. At later time, the boundary condition changes and the energy density is no more maintain to zero. The counterterm then stays equal to its value at time

$2\pi R/c$ (which we note CT), obtained from the transition condition $\rho(2\pi R/c) = 0$

$$\frac{5}{R} \int_{\omega_n(k)>0} (\dots) - \frac{5}{R} \int_{\omega_n(k)<c/R} (\dots) + CT = 0 \quad (15)$$

In the late time regime, $t \gg R/c$, the cut off introduced by the Hubble length can be neglected so that the present day energy density ρ_0 reads:

$$\begin{aligned} \rho_0 &= \frac{5}{R} \int_{\omega_n(k)>0} \frac{d^3 k}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \frac{1}{2} \sqrt{k^2 + n^2/R^2} + CT \\ &= \frac{5}{R} \int_{\omega_n(k)<c/R} \frac{d^3 k}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \frac{1}{2} \sqrt{k^2 + n^2/R^2} \end{aligned} \quad (16)$$

after using equation (15). We see in the previous expression that R acts as an UV cutoff for the sum of zero point energies. The condition $\omega_n(k)^2 < (1/R)^2$ implies that only the term $n = 0$ contributes to the last integral, rendering it elementary. This allows us to obtain the value of the density (reintroducing explicitly \hbar and c) :

$$\rho_0 = \frac{5\hbar c}{32\pi^3 R^5} \quad (17)$$

The other components of the energy-momentum tensor can be obtained from ρ_0 . Indeed, the traceless nature of the gravitational field together with the symmetry of the problem requires that

$$\langle T^{\mu\nu} \rangle = \rho_{\text{cas}} (g^{\mu\nu} + 5\hat{n}^\mu \hat{n}^\nu) \quad (18)$$

with \hat{n}^μ the unit spacelike vector pointing along the extra dimension ($\hat{n}^2 = -1$) and ρ_{cas} is a constant (because of the conservation laws $\partial_\mu T^{\mu\nu} = 0$). One finds that the pressure along the extra dimension (perpendicular to the brane) is $p^\perp = 4\rho_0$, while the pressure in the brane (the usual spacetime dimension) is such that $p^\parallel = -\rho_0$. This situation is analogous to the previous discussion on the electromagnetic Casimir situation (section 2.). Note also that p^\perp could have been derived from energy conservation when considering a variation of the radius R . On the brane, the energy-momentum tensor is obtained by integrating along the fifth dimension,

$$\rho_{\text{brane}} = \frac{5\hbar c}{16\pi^2 R^4} \quad , \quad p_{\text{brane}} = -\rho_{\text{brane}} \quad (19)$$

Equation (19) can thus be identified with the present day dark energy density $\rho_{\text{DE}} = 0,7\rho_c \approx 4 \text{ keV/cm}^3$ for an appropriate value of R . Such an identification leads to a prediction for the size of the extra dimension given by

$$R = \left(\frac{5\hbar G}{2\pi c^3 \Lambda} \right)^{\frac{1}{4}} = 35 \text{ } \mu\text{m} \quad (20)$$

A consequence of the present discussion is that gravitational laws are modified at the scale of the radius R , which is precisely the range of present experimental apparatus, such as experiments testing the gravitational force (Kapner et al. 2007; Adelberger et al. 2009) or experiments aiming at measuring the Casimir force (Antoniadis et al. 2011). More interestingly, because of numerical prefactors, the value of R predicted here is slightly smaller than the dimensional length scale ℓ_Λ introduced in the introduction.

The four-dimensional gravitational potential, in the presence of one extra dimension is obtained as an infinite sum of Yukawa potentials, each of them corresponding the one massive mode of the Kaluza Klein tower (Arkani-Hamed et al. 1999; Kehagias & Sfetsos 2000)

$$V = -\frac{G_3 M}{r} \sum_{m=-\infty}^{\infty} e^{-|m|\frac{r}{R}} = -\frac{G_3 M}{r} \coth\left(\frac{r}{2R}\right) \quad (21)$$

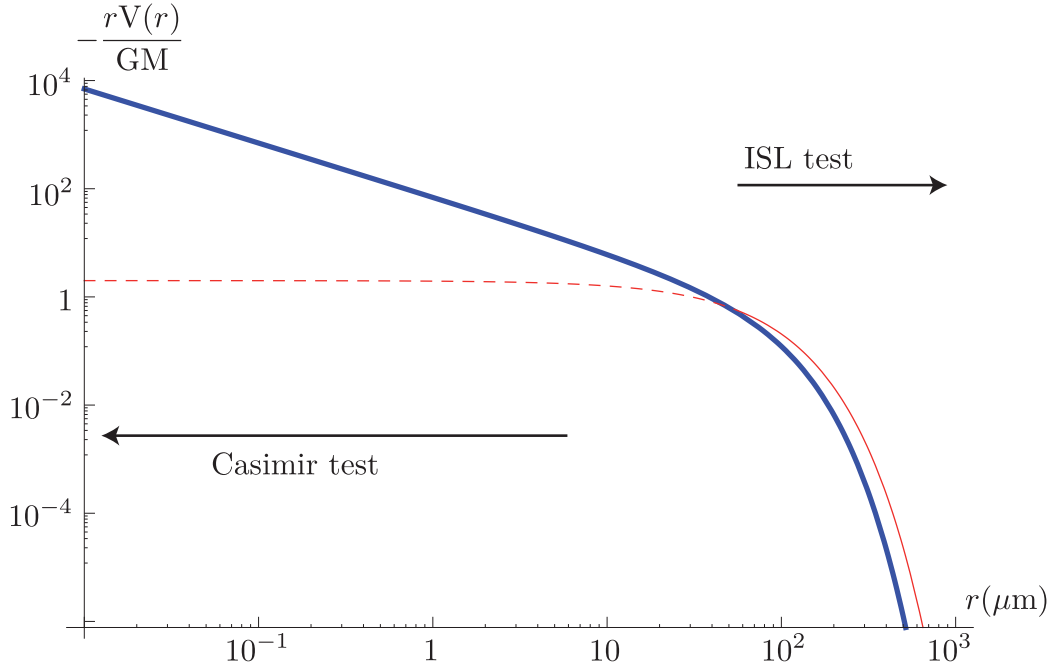


Fig. 1. Point-particle gravitational potential for one extra dimension (bold black line) and Yukawa (red line). The Yukawa modification is taken with a range given by the present day constraint stemming from Adelberger et al. (2009), $\lambda = 44 \mu\text{m}$. The dashed part of this curve corresponds to scales not tested in Adelberger et al. (2009). The extradimension potential is plotted for a value of the radius given by $R = 35 \mu\text{m}$. Our prediction is not excluded by experiments, but further improvement will soon give a definite answer. The short scale behavior is different from the pure Yukawa modification usually searched for in experiments.

For $r \gg R$, the previous expression is given by the Newtonian expression ($m = 0$) plus the contribution of the lightest Kaluza-Klein modes ($n = \pm 1$),

$$V \simeq -\frac{G_3 M}{r} (1 + 2 \exp(-r/R)) \quad , \quad r \gg R \quad (22)$$

It corresponds to a Yukawa modification with strength $\alpha = 2$ and a range given by the radius R of the extra dimension. Using this type of potential, the analyses of ISL tests (Adelberger et al. 2009) give at 95% confidence level a maximum size of $44 \mu\text{m}$ for R . Our prediction is therefore just below the present-day limits. Improvement on these measurements will therefore be critical in order to test our model. Note nevertheless that when probing the ISL at distance $\sim R$, the complete expression should be used instead of the simple Yukawa description (see figure 1). At smaller scales, the best constraints on gravity laws are obtained by Casimir force measurements (Decca et al. 2007). The experiments are performed at distance smaller than the size of the extra dimension, leading to a different behavior for the potential (21)

$$V \simeq -\frac{G_3 M}{r} \left(1 + \frac{2R}{r} \right) \quad (23)$$

It leads to a power-law modification of the gravitational force between two test masses with an amplitude scaled by R given by eq. (20). This modification could be searched for in Casimir experiments operating at small distances, although present-day limits in those experiments are still several orders of magnitude above our prediction (Antoniadis et al. 2011).

4. Conclusion

The zero-point energy from a quantized field present in additional compact dimension naturally provides a non-vanishing value for the vacuum contribution to density of the universe, through a Casimir like effect. Such a term is naturally Lorentz invariant in the usual 4D spacetime and

therefore may provide a natural explanation for the observed cosmological constant. However, present day experimental limits on possible additional dimension, summarized in Table 1., exclude more than one extra dimension for such contribution to be of the order of the observed dark energy density ¹. The case of one extra dimension is still allowed, although in the case of a Minkowski space-time it leads to a negative contribution to the density of the universe. In the cosmological context, we have proposed that the Hubble radius acts as a cut-off to mode wavelenths contributing to the vacuum expectation value, i.e. an infrared cut-off in 4. Using a zero-point energy calculation, we showed that this infrared cut-off leads to a positive contribution. Therefore this mechanism provides a natural explanation for the origin of the observed cosmic acceleration which appears as a manifestation of the quantized gravitational field in an additional dimension. A first consequence of this model is that the Planck energy scale is lowered to $\sim 10^9$ GeV. A second consequence is that the equation of state of cosmological dark energy should be exactly that of a cosmological constant, i.e. $w = -1$. A third consequence is that gravitation law would be modified on scales of the order of the size of the compact dimension, which is $35 \mu\text{m}$, a value below the purely dimensional Dark Energy length scale $(\hbar c/\rho_v)^{1/4} \sim 85 \mu\text{m}$ (Kapner et al. 2007; Beane 1997) and below but close to present experimental limits on departure to the inverse square law of gravity law at short scales. This leaves open the fascinating possibility that tests of the gravitation law on short distance shed new light on the nature and origin of cosmic acceleration.

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References

- Adelberger, E. G., Gundlach, J. H., Heckel, B. R., Hoedl, S., & Schlamminger, S. 2009, *Progress in Particle and Nuclear Physics*, 62, 102
- Amendola, L., Appleby, S., Bacon, D., et al. 2012, *ArXiv e-prints*
- Antoniadis, I., Baessler, S., Büchner, M., et al. 2011, *Comptes Rendus Physique*, 12, 755
- Appelquist, T. & Chodos, A. 1983, *Phys. Rev. D*, 28, 772
- Appelquist, T. & Chodos, A. 1983, *Physical Review Letters*, 50, 141
- Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. 1998, *Physics Letters B*, 429, 263
- Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. 1999, *Phys. Rev. D*, 59, 086004
- Beane, S. R. 1997, *General Relativity and Gravitation*, 29, 945
- Beringer, J., Arguin, J.-F., Barnett, R. M., et al. 2012, *Phys. Rev. D*, 86, 010001
- Blanchard, A. 2010, *The astronomy and astrophysics review*, 18, 595
- Brown, L. S. & Maclay, G. J. 1969, *Physical Review*, 184, 1272
- Cahill, K. 2011, *ArXiv e-prints*
- Candelas, P. & Weinberg, S. 1984, *Nuclear Physics B*, 237, 397
- Chodos, A. & Myers, E. 1985, *Phys. Rev. D*, 31, 3064
- Clifton, T., Ferreira, P. G., Padilla, A., & Skordis, C. 2012, *Physics Reports*, 513, 1
- Cognola, G., Elizalde, E., & Zerbini, S. 2005, *Physics Letters B*, 624, 70
- Decca, R. S., López, D., Fischbach, E., et al. 2007, *European Physical Journal C*, 51, 963
- Einstein, A. 1917, *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin)*, Seite 142-152., 142
- Elizalde, E. 2006a, in *American Institute of Physics Conference Series*, Vol. 878, *The Dark Side of the Universe*, ed. C. Manoz & G. Yepes, 232–239
- Elizalde, E. 2006b, *Journal of Physics A Mathematical General*, 39, 6299
- Ellis, G. F. R., van Elst, H., Murugan, J., & Uzan, J.-P. 2011, *Classical and Quantum Gravity*, 28, 225007
- Frieman, J. A., Turner, M. S., & Huterer, D. 2008, *Annual review of astronomy and astrophysics*, 46, 385
- Hannestad, S. & Raffelt, G. G. 2003, *Phys. Rev. D*, 67, 125008
- Hollenstein, L., Jaccard, M., Maggiore, M., & Mitsou, E. 2011, *ArXiv e-prints*
- Kaluza, T. 1983, in *Unified Field Theories of >4 Dimensions*, ed. V. De Sabbata & E. Schmutzer, 427
- Kantowski, R. & Milton, K. A. 1987, *Phys. Rev. D*, 36, 3712
- Kapner, D. J., Cook, T. S., Adelberger, E. G., et al. 2007, *Phys. Rev. Lett.*, 98, 021101
- Kehagias, A. & Sfetsos, K. 2000, *Physics Letters B*, 472, 39
- Kragh, H. 2011, *ArXiv e-prints*
- Laureijs, R., Amiaux, J., Arduini, S., et al. 2011, *ArXiv e-prints*

¹ Strictly speaking this conclusion holds only for odd number of extra dimensions, as the actual Casimir contribution for even number of extra dimensions is not properly known.

- Lemaître, G. 1934, Proceedings of the National Academy of Science, 20, 12
- LSST Science Collaborations, Abell, P. A., Allison, J., et al. 2009, ArXiv e-prints
- Martin, J. 2012, Comptes Rendus Physique, 13, 566
- Milton, K. 2001, The Casimir Effect: Physical Manifestations of Zero-Point Energy (World Scientific)
- Myers, E. 1986, Phys. Rev. D, 33, 1563
- Peebles, P. J. & Ratra, B. 2003, Reviews of Modern Physics, 75, 559
- Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, Astrophysical Journal, 517, 565
- Randall, L. & Sundrum, R. 1999, Phys. Rev. Lett., 83, 3370
- Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, Astron. J., 116, 1009
- Rohrlich, D. 1984, Phys. Rev. D, 29, 330
- Rubakov, V. A. 2001, Physics Uspekhi, 44, 871
- Smolin, L. 2009, Phys. Rev. D, 80, 084003
- Straumann, N. 2002, ArXiv General Relativity and Quantum Cosmology e-prints
- Tsujikawa, S. 2010, in Lecture Notes in Physics, Berlin Springer Verlag, ed. G. Wolschin, Vol. 800, 99–145
- Weinberg, S. 1989, Reviews of Modern Physics, 61, 1
- Zel'Dovich, Y. B. 1967, Soviet Journal of Experimental and Theoretical Physics Letters, 6, 316
- Zel'dovich, Y. B. 1968, Soviet Physics Uspekhi, 11, 381

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